2. Reduced echelon form: a. Echelon form: b and d. Not echelon: c.

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -34 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & -8 & -16 & -34 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -10 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} ① & 0 & -1 & 0 \\ 0 & ① & 2 & 0 \\ 0 & 0 & 0 & ① \end{bmatrix}.$$ Pivot cols 1, 2, and 4

$$\begin{bmatrix} ① & 3 & 5 & 7 \\ 3 & ⑤ & 7 & 9 \\ 5 & 7 & 9 & ① \end{bmatrix}$$

6. $$\begin{bmatrix} ■ & * \\ 0 & ■ \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} ■ & * \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & ■ \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

0. $$\begin{bmatrix} 1 & -2 & -1 & 3 \\ 3 & -6 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 0 & 1 & -7 \end{bmatrix} \sim \begin{bmatrix} ① & -2 & 0 & -4 \\ 0 & 0 & ① & -7 \end{bmatrix}$$

Corresponding system: $\begin{aligned} ㊀ - 2x_2 &= -4 \\ ㊂ &= -7 \end{aligned}$

Basic variables: x_1, x_3; free variable: x_2. General solution: $\begin{cases} x_1 = -4 + 2x_2 \\ x_2 \text{ is free} \\ x_3 = -7 \end{cases}$

14. $$\begin{bmatrix} 1 & 2 & -5 & -6 & 0 & -5 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} ① & 0 & 7 & 0 & 0 & -9 \\ 0 & ① & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & ① & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Corresponding system: $\begin{aligned} ㊀ + 7x_3 &= -9 \\ ㊁ - 6x_3 - 3x_4 &= 2 \\ ㊄ &= 0 \\ 0 &= 0 \end{aligned}$

Basic variables: x_1, x_2, x_5; free variables: x_3, x_4. General solution: $\begin{cases} x_1 = -9 - 7x_3 \\ x_2 = 2 + 6x_3 + 3x_4 \\ x_3 \text{ is free} \\ x_4 \text{ is free} \\ x_5 = 0 \end{cases}$

16. a. The system is consistent, with a unique solution.

b. The system is consistent. There are many solutions because x_2 is a free variable.

20. $$\begin{bmatrix} 1 & 3 & 2 \\ 3 & h & k \end{bmatrix} \sim \begin{bmatrix} ① & 3 & 2 \\ 0 & h-9 & k-6 \end{bmatrix}$$

a. When $h = 9$ and $k \neq 6$, the system is inconsistent, because the augmented column is a pivot column.

b. When $h \neq 9$, the system is consistent and has a unique solution. There are no free variables.

c. When $h = 9$ and $k = 6$, the system is consistent and has many solutions.

30. Example: $\begin{aligned} x_1 + x_2 + x_3 &= 4 \\ 2x_1 + 2x_2 + 2x_3 &= 5 \end{aligned}$